Pearson Edexcel Level 3

GCE Mathematics

Advanced Subsidiary

Paper 1: Pure

Monday 27 May 2019 **Time: 1 hour 45 minutes**	**Paper Reference(s)** **8MA0/01**

You must have:
Mathematical Formulae and Statistical Tables, calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 16 questions in this paper. The total mark is 87.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1.

The line L_1 has equation $4x + 2y - 3 = 0$

(a) Find the gradient of L_1.

(1)

The line L_2 is perpendicular to L_1 and passes through the point (2, 5).

(b) Find the equation of L_2 in the form $y = mx + c$, where m and c are constants.

(3)

a) L_1 $4x + 2y = 3$

$2y = 3 - 4x$

$y = \frac{3}{2} - 2x$

$\therefore M_{L_1} = -2$ B1

b) $\therefore$ $M_{L_2} = -\frac{1}{-2} = \frac{1}{2}$ B1

$\frac{y-5}{x-2} = \frac{1}{2}$ M1 A1 oe.

$y - 5 = \frac{1}{2}(x - 2)$

$y = \frac{1}{2}x + 4$

Total for Q1 is 4 marks

2.

Find, using algebra, all real solutions to the equation

$$a^4 - 2a^2 - 80 = 0$$

(4)

let $b = a^2$

$b^2 - 2b - 80 = 0$

$(b-10)(b+8) = 0$ M1A1 $\times -80$, $+ -2$: $-10, 8$

$b = 10$ $b = -8$

$\Rightarrow a^2 = 10$ $a^2 = -8$

$a = \pm\sqrt{10}$ no solutions M1

$a = \sqrt{10}$, $a = -\sqrt{10}$ A1 (both and no more)

Total for Q2 is 4 marks

3.

$$\frac{dy}{dx} = -x^3 + \frac{4x-5}{2x^3}, x \neq 0$$

Given that $y = 7$ at $x = 1$, find y in terms of x, giving each term in its simplest form.

(6)

$y = \int -x^3 + \frac{4x-5}{2x^3} \, dx$

$y = \int -x^3 + 2x^{-2} - \frac{5}{2}x^{-3} \, dx$ M1

raise 1 power

$y = \frac{-x^4}{4} - 2x^{-1} + \frac{5}{4}x^{-2} + C$ M1 A1 A1

any 2 correct all correct

$x=1$ $y=7$ $7 = -\frac{1}{4} - 2 + \frac{5}{4} + C$ M1

$7 = -2 + 1 + C$

$8 = C$

$y = -\frac{x^4}{4} - 2x^{-1} + \frac{5}{4}x^{-2} + 8$ A1

Total for Q3 is 6 marks

4.

A large plant pot shown in figure A initially has some water in it. Jim then starts to pour more water into it at a constant rate.

The depth of water, D cm, was measured t seconds after Jim started pouring more water in.

Exactly 5 seconds after Jim starts pouring, the depth was 13.52cm
Exactly 8 seconds after he started pouring the depth was 18.68cm

Figure A

Using a **linear** model

(a) Find an equation for D in terms of t (3)

initial depth

(b) Find the depth of the water that was already in the plant pot before Jim started filling it. (1)

Explain why

(c) Explain why a linear model isn't suitable to model the depth over time (1)

a) $D = mt + c$

$t=5 \quad D=13.52 \quad (1)\ 13.52 = 5m + c$ B1 for equations

$t=8 \quad D=18.68 \quad (2)\ 18.68 = 8m + c$

$(2)-(1) \quad 5.16 = 3m$ M1

$m = 1.72$

$(1) \quad 13.52 - 5\times1.72 = c$

$c = 4.92\text{cm}$

$\therefore \quad D = 1.72t + 4.92$ A1

b) 4.92 cm B1 (must have units

c) Plant pot wider further up, depth wouldn't increase at constant rate B1

or $\frac{dD}{dt}$ would be decreasing oe.

Total for Q4 is 5 marks

5.
A curve has equation

$$y = \sqrt{x} + \frac{4}{x} + 4, \qquad x > 0$$

(a) Find, in simplest form $\frac{dy}{dx}$ (3)

(b) Hence find the ~~exact~~ range of values of x for which the curve is increasing (2)

a) $y = x^{1/2} + 4x^{-1} + 4$ B1

$\frac{dy}{dx} = \frac{1}{2}x^{-1/2} - 4x^{-2}$ M1 A1

× power, decrease power by 1

b) $\frac{dy}{dx} \geq 0$

allow > or ≥ throughout as per Edexcel guidance

$\frac{1}{2}x^{-1/2} - 4x^{-2} \geq 0$ M1

$\frac{1}{2x^{1/2}} \geq \frac{4}{x^2}$

$x^{3/2} \geq 8$

$\sqrt{x^3} \geq 8$

$x^3 \geq 64$

$x \geq 4$ A1

Total for Q5 is 5 marks

6.

Prove by first principles that $\frac{d}{dx}(x^2) = 2x$

(3)

Let $f(x) = x^2$

$$f'(x) = \lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{x + h - x} \right)$$

$$f'(x) = \lim_{h \to 0} \left(\frac{(x+h)^2 - x^2}{h} \right)$$ B1 RHS only

$$f'(x) = \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}$$ M1

$$f'(x) = \lim_{h \to 0} 2x$$ A1 no errors with correct notation on LHS and limits on RHS (may use $\frac{dy}{dx}$)

$$f'(x) = 2x$$

$$\therefore \frac{d}{dx}(x^2) = 2x$$

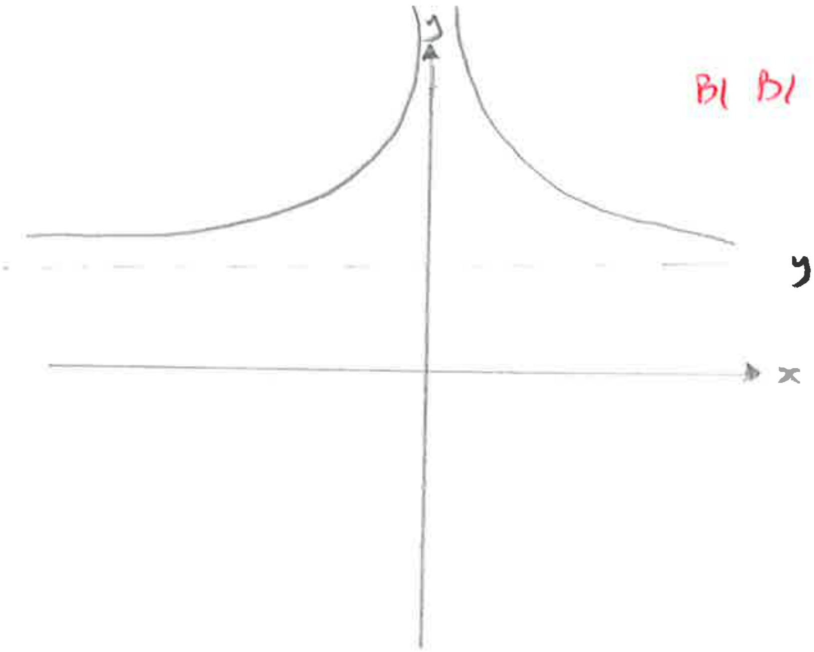
Total for Q6 is 3 marks

7.
The curve C has equation

$$y = \frac{k}{x^2} + 2, \quad x \in \mathbb{R}, x \neq 0$$

Where k is a constant.
(a) Sketch C in the space below, stating the horizontal asymptote

(3)



B1 B1 for each section

$y = 2$ B1

The line l has equation $y = -2x + 5$

(b) Show that the x coordinate of any point of intersection of l with C is given by a solution of the equation

$$k - 3x^2 + 2x^3 = 0$$

(2)

(b) $y = \frac{k}{x^2} + 2 \qquad y = -2x + 5$

$\Rightarrow \frac{k}{x^2} + 2 = -2x + 5$ *

needed for A1

$\frac{k}{x^2} = -2x + 3$ *

$k = -2x^3 + 3x^2$ M1

$k + 2x^3 - 3x^2 = 0$

$k - 3x^2 + 2x^3 = 0$ A1 no errors.

Total for Q7 is 5 marks

8.

(a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$\left(3+\frac{4x}{5}\right)^5$$

~~(4)~~ (3)

(b) Explain how you could use your expansion to estimate the value of 2.96^5.
You do not need to perform this calculation

~~(1)~~ (2)

(a) $^5C_0(3^5)\left(\frac{4x}{5}\right)^0 + {}^5C_1(3^4)\left(\frac{4x}{5}\right)^1 + {}^5C_2(3^3)\left(\frac{4x}{5}\right)^2$

M1 use of one of these A1 all of them correct.

$= 243 + 5\times 81 \times \frac{4x}{5} + 10\times 27\times \frac{16x^2}{25}$

may be implied by correct answer

$= 243 + 324x + \frac{864x^2}{5}$ ~~A1~~

A1 all simplified

(b) $2.96 = 3 + \frac{4x}{5}$

$\Rightarrow \quad x = -\frac{1}{20}$ B1

Substitute $x = -\frac{1}{20}$ into the expansion found in (a)
B1

Total for Q8 is 5 marks

9.

On 31st December 2014 the UK government implemented a charge on plastic bags in an attempt to reduce the rising use of these bags and the environmental impact that this has.

n years after the charge was implemented, the government targeted for the number of plastic bags used to be modelled by the following equation

$$x = 7.6 - 0.2(n-2)^2$$

Where x is the number of plastic bags, in billions, being used in the preceding year.

(a) Calculate, according to the model, how many bags were used in the year ending 31st December 2014.

(1)

(b) Find

(i) The year when the bag usage was targeted to be at its highest

(ii) How many bags were predicted to be used in the year found in (i)

(2)

(c) In the year ending the 31st December 2018 seven billion bags were used. Evaluate whether the government were meeting their target at this point, justifying your answer.

(2)

(d) Explain why the model will not be valid in 2025.

(1)

(a) When $n=0$

$x = 7.6 - 0.2(-2)^2$

$x = 6.8$

6.8 billion bags B1 (in context)

(b) (i) When $0.2(n-2)^2 = 0 \Rightarrow n=2$

2016 B1

(ii) 7.6 billion B1 (in context: only penalise lack of context once from (a) and b(ii))

c) 2018 $\Rightarrow n=4$

$x = 7.6 - 0.2(4-2)^2$

$x = 6.8$ billion B1

Should be at 6.8 billion < 7 billion so no B1 with comparison

d) 2025 $\Rightarrow$ $n = 11$ (allow 10)

$x = 7 \cdot 6 - 0 \cdot 2 (11 - 2)^2$ *

$x = -8 \cdot 6$

Can't have negative amount of bags used

$\Rightarrow$ model not suitable B1

Total for Q6 is 6 marks

10.

A circle C has equation

$$x^2 + y^2 + 6x - 4y + 5 = 0$$

(a) Find

(i) The coordinates of the centre of C,

(ii) The exact radius of C

(3)

The straight line, with equation $x = k$, where $k$is a constant, ~~cuts~~ intersects C at 2 distinct point.

(b) Find the possible values for k.

(2)

(a) (i) & (ii)

$$x^2 + 6x \quad + y^2 - 4y \quad + 5 = 0$$

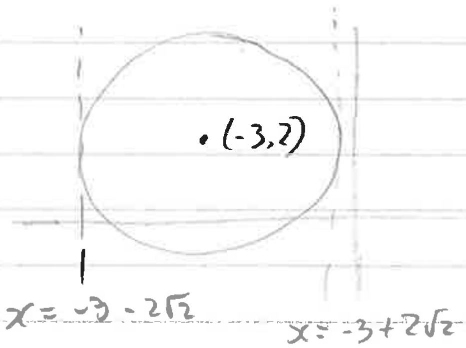
$$(x+3)^2 - 9 \quad + (y-2)^2 - 4 \quad + 5 = 0$$ M1 complete square.

$$(x+3)^2 + (y-2)^2 = 8 \quad (2\sqrt{2})^2$$

(i) Centre at $(-3, 2)$ B1

(ii) Radius $2\sqrt{2}$ B1

b)



$$\boxed{\text{or use of } b^2 - 4ac > 0}$$

$-3 - 2\sqrt{2} < k < -3 + 2\sqrt{2}$ B1 B1

~~-0.172~~ -0.172

-5.83

at least one end correct

all correct

or

both and $\leq$

Allow decimals (at least 3SF)

Total for Q10 is 5 marks

11.

$$f(x) = x^3 + x^2 - 8x - 12$$

(a) Prove that $(x - 3)$ is a factor of $f(x)$.

(2)

(b) Hence, using algebra, show that the equation $f(x) = 0$ has only 2 distinct roots.

(3)

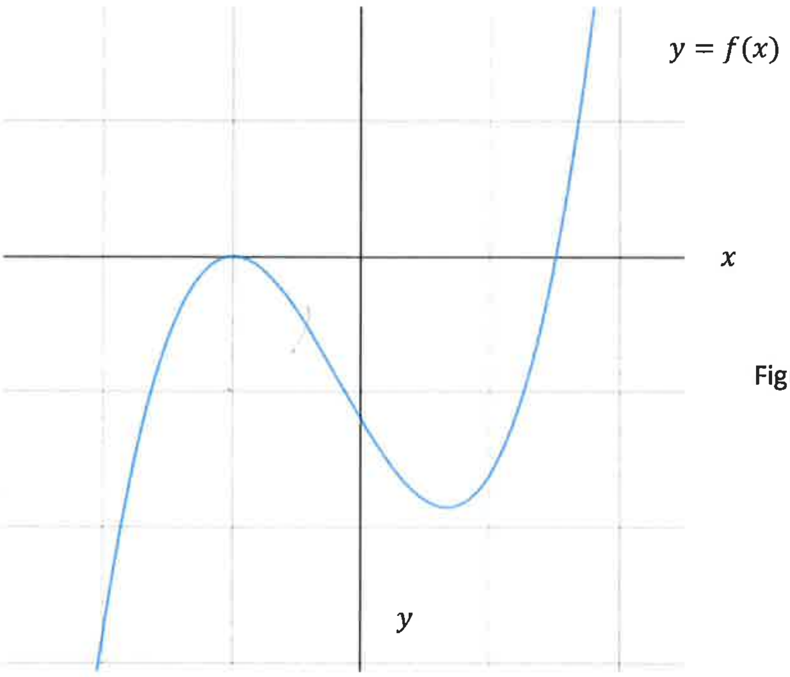


Figure B

Figure B shows a sketch of part of the curve $y = f(x)$.

(c) Deduce, giving reasons for your answer, the number of real roots of the equation

$$2x^3 + 2x^2 - 16x - 50 = 0$$

(3)

Given that k is a constant and the curve with equation $y = f(kx)$ passes through (1,0),

(d) Find the two possible values of k

(2)

(a) $f(3) = 3^2 + 3^2 - 8 \times 3 - 12$ B1 most show
$= 0$

$\Rightarrow (x-3)$ is a factor B1 linking $f(3)=0$
by the factor theorem and $(x-3)$
as factor

b) $$\begin{array}{r} x^2 + 4x + 4 \\ x-3 \overline{\smash{)}\, x^3 + x^2 - 8x - 12} \\ -(x^3 - 3x^2) \\ \hline 4x^2 - 8x \\ -(4x^2 - 12x) \\ \hline 4x - 12 \\ 4x - 12 \\ \hline 0 \end{array}$$

M1 A1

at least one step correct

$f(x) = (x-3)(x^2 + 4x + 4)$

$f(x) = (x-3)(x+2)^2$ B1

$\therefore$ roots at $x=3$ and $x=-2$

or first 2 marks

$(x-3)(ax^2 + bx + c) = x^3 + x^2 - 8x - 12$

$x^3 \Rightarrow \quad a = 1$

Constant $\Rightarrow \quad -3c = -12$

$c = 4$

$x \Rightarrow \quad c - 3b = -8$

or $\quad -4 - 8 = -3b$

$b = 4$ M1

$x^2 \Rightarrow \quad -3a + b = 1$

$b = 1 + 3$

$b = 4$

$f(x) = (x-3)(x^2 + 4x + 4)$ A1

c) $2x^3 + 2x^2 - 16x - 50 = 0$

$\Rightarrow x^3 + x^2 - 8x - 25 = 0$

$\Rightarrow x^3 + x^2 - 8x - 12 = 13$ M1

where $y = 13$ meets $f(x) \Rightarrow$ 1 real root B1

(d) $f(kx)$ is stretch SF $1/k$ in x direction

$3 \to 1$ stretch of $1/3 \Rightarrow k = 3$ B1

$-2 \to 1$ stretch of $-1/2 = k = -2$ B1

Total for Q11 is 10 marks

12.

Show that

$$\frac{10cos^2x - sinx - 7}{2sinx - 1} \equiv -3 - 5sinx$$

(4)

LHS

$$\frac{10\cos^2 x - \sin x - 7}{2\sin x - 1}$$

M1 use of $\sin^2 x + \cos^2 x = 1$

$$= \frac{10(1 - \sin^2 x) - \sin x - 7}{2\sin x - 1}$$

$$= \frac{10 - 10\sin^2 x - \sin x - 7}{2\sin x - 1}$$

$$= \frac{-10\sin^2 x - \sin x + 3}{2\sin x - 1} \quad \text{A1}$$

$$= \frac{-(10\sin^2 x + \sin x - 3)}{2\sin x - 1}$$

$\times -30$ } $6, -5$
$+ 1$ ✓

$$= \frac{-(10\sin^2 x - 5\sin x + 6\sin x - 3)}{2\sin x - 1}$$

$$= \frac{-(5\sin x(2\sin x - 1) + 3(2\sin x - 1))}{2\sin x - 1}$$

$$= \frac{-\cancel{(2\sin x - 1)}(5\sin x + 3)}{\cancel{(2\sin x - 1)}} \quad \text{A1} \quad \text{oe}$$

$$= -5\sin x - 3$$

$$\therefore \frac{10\cos^2 x - \sin x - 7}{2\sin x - 1} = -3 - 5\sin x$$

A1
all steps
shown
clearly

Total for Q12 is 4 marks

13.

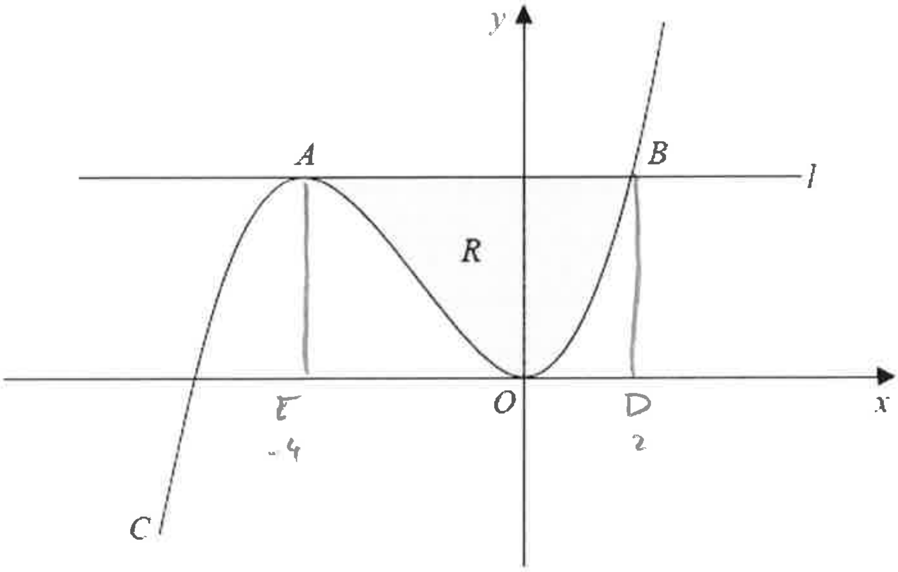


Figure D

Figure D shows a sketch of part of the curve C with equation

$$y = \frac{1}{8}x^3 + \frac{3}{4}x^2, \qquad x \in \mathbb{R}$$

The curve C has a maximum turning point at the point A and a minimum turning point at the origin O.

The line l touches the curve C at the point A and cuts the curve C at the point B.

The x coordinate of A is −4 and the x coordinate of B is 2.

The finite region R, shown shaded in Figure 3, is bounded by the curve C and the line l.

Use integration to find the area of the finite region R.

(7)

$x = 2$ $\quad y = \frac{1}{8}2^3 + \frac{3}{4}2^2$

$y = 1 + 3$

$y = 4$

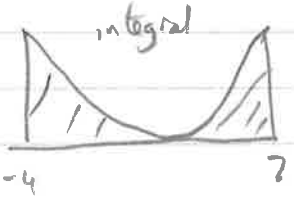
or $x = -4$ $\quad y = 4$

Area of rectangle ABDE $= 4 \times (2 - -4)$

$= 24$ $\quad$ B1

R = Rectangle

−



$$R = 24 - \int_{-4}^{2} \frac{1}{8}x^3 + \frac{3}{4}x^2 \, dx$$

$$R = 24 - \left[\frac{1}{4}\frac{1}{8}x^4 + \frac{1}{3}\frac{3}{4}x^3\right]_{-4}^{2}$$

M1 $x^n \to x^{n+1}$

A1 one term correct

$$R = 24 - \left(\underbrace{\left(\frac{1}{32}\times 2^4 + \frac{1}{4}2^3\right) - \left(\frac{1}{32}(-4)^4 + \frac{1}{4}(-4)^3\right)}_{M1}\right)$$

$$R = 24 - \frac{21}{2}$$

A1 for $\frac{21}{2}$

M1 for "24" $-$ "$\frac{21}{2}$"

$$R = \frac{27}{2}$$

A1

Total for Q13 is 7 marks

14.

The mass, m grams, of a leaf t days after it has been picked from a tree is given by

$$m = pe^{-kt}$$

where k and p are positive constants.

When the leaf is picked from the tree, its mass is 7.5 grams and 4 days later its mass is 2.5 grams.

(a) Show that $k = \frac{1}{4}\ln(3)$

(4)

(b) Find the value of t when $\frac{dm}{dt} = -0.6\ln(3)$

(5)

(a) when $t=0$ $m=7.5$ $\Rightarrow$ $p=7.5$ B1

$t=4$ $m=2.5$

$2.5 = 7.5e^{-4k}$ M1

$\frac{1}{3} = e^{-4k}$

$\ln\frac{1}{3} = -4k$ M1

$\ln 1 - \ln 3 = -4k$

$\ln 3 = 4k$

$k = \frac{1}{4}\ln 3$ A1 (convincing steps from M1 & A1)

b) $m = \frac{15}{2}e^{-\frac{1}{4}(\ln 3)t}$

$\frac{dm}{dt} = \frac{15}{2}\left(-\frac{1}{4}\ln(3)\right)e^{-\left(\frac{1}{4}\ln 3\right)t}$ M1 A1 ft with p & k

power same multiply by k (ignore anything else)

$$\Rightarrow \quad -0.6 \ln 3 = -\frac{15}{8} \ln 3 \; e^{-\left(\frac{1}{4}\ln 3\right)t}$$

$$0.32 = e^{-\left(\frac{1}{4}\ln 3\right)t} \qquad \text{M1}$$

$$\ln 0.32 = -\left(\frac{1}{4}\ln 3\right)t \qquad \text{M1}$$

$$\frac{\ln 0.32}{-\frac{1}{4}\ln 3} = t$$

$$t = 4.148\ldots$$

$$t = 4.15 \text{ days} \qquad \text{A1} \qquad \text{awrt } 4.1 \underline{\text{days}}$$

needed.

Total for Q14 is 9 marks

15.

Prove that $x^2 + 8x + 10 \geq 2x + 1$ $x \in \mathbb{R}$

(4)

LHS

$x^2 + 8x + 10$

$= x^2 + 6x + 9 + 2x + 1$ B1 split up

$= \underbrace{(x+3)^2 \cancel{-9+9}}_{\geq 0} + 2x + 1$ M1

B1 state $(x+3)^2 \geq 0$

$\Rightarrow (x+3)^2 + 2x + 1 \geq 2x + 1$ A1 no errors.

$\Rightarrow x^2 + 8x + 10 \geq 2x + 1$

note

$x^2 + 8x + 10 \geq 2x + 1$

$\Rightarrow x^2 + 6x + 9 \geq 0$

$\Rightarrow \underbrace{(x^2 + 3)^2}_{\geq 0} \geq 0$

$\Rightarrow x^2 + 8x + 10 \geq 2x + 1$

or scores

B0 M1 B1 A0.

note 2

Consider $(x+3)^2 \geq 0$ B1

$x^2 + 6x + 9 \geq 0$

$x^2 + 8x + 10 - (2x+1) \geq 0$ M1 B1.

$\therefore$ $x^2 + 8x + 10 \geq 2x + 1$ A1

scores B1 M1 B1 A1

Total for Q15 is 4 marks

16.

(a) Two non zero vectors, **a** and **b**, are such that

$$|\boldsymbol{a}+\boldsymbol{b}| = |\boldsymbol{a}| - |\boldsymbol{b}|$$

Explain, geometrically, the significance of this statement.

(1)

(b) Two different vectors, $\boldsymbol{m}$ and $\boldsymbol{n}$, are such that $|\boldsymbol{m}| = 4$ and $|\boldsymbol{m}-\boldsymbol{n}| = 6$.

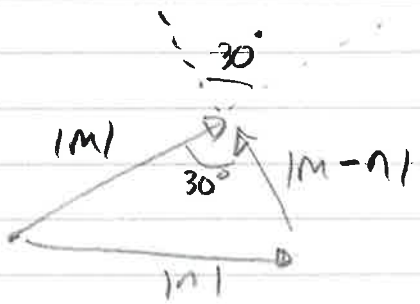
The angle between $\boldsymbol{m}$ and $\boldsymbol{m}-\boldsymbol{n}$ is 30^o.

Find $|\boldsymbol{n}|$

(4)

(a) a and b are in opposite directions B1

(b)



B1 may be implied by correct cosine rule.

$\Rightarrow \quad a^2 = b^2 + c^2 - 2bc\cos A$

M1 use of

$|n|^2 = |m|^2 + |m-n|^2 - 2|m||m-n|\cos 30$

$|n|^2 = 4^2 + 6^2 - 2 \times 4 \times 6 \dfrac{\sqrt{3}}{2}$ A1

$|n|^2 = 52 - 24\sqrt{3}$

$|n| = 3.23$ (3SF) A1 awrt 3.23

Total for Q15 is ~~4~~ 5 marks